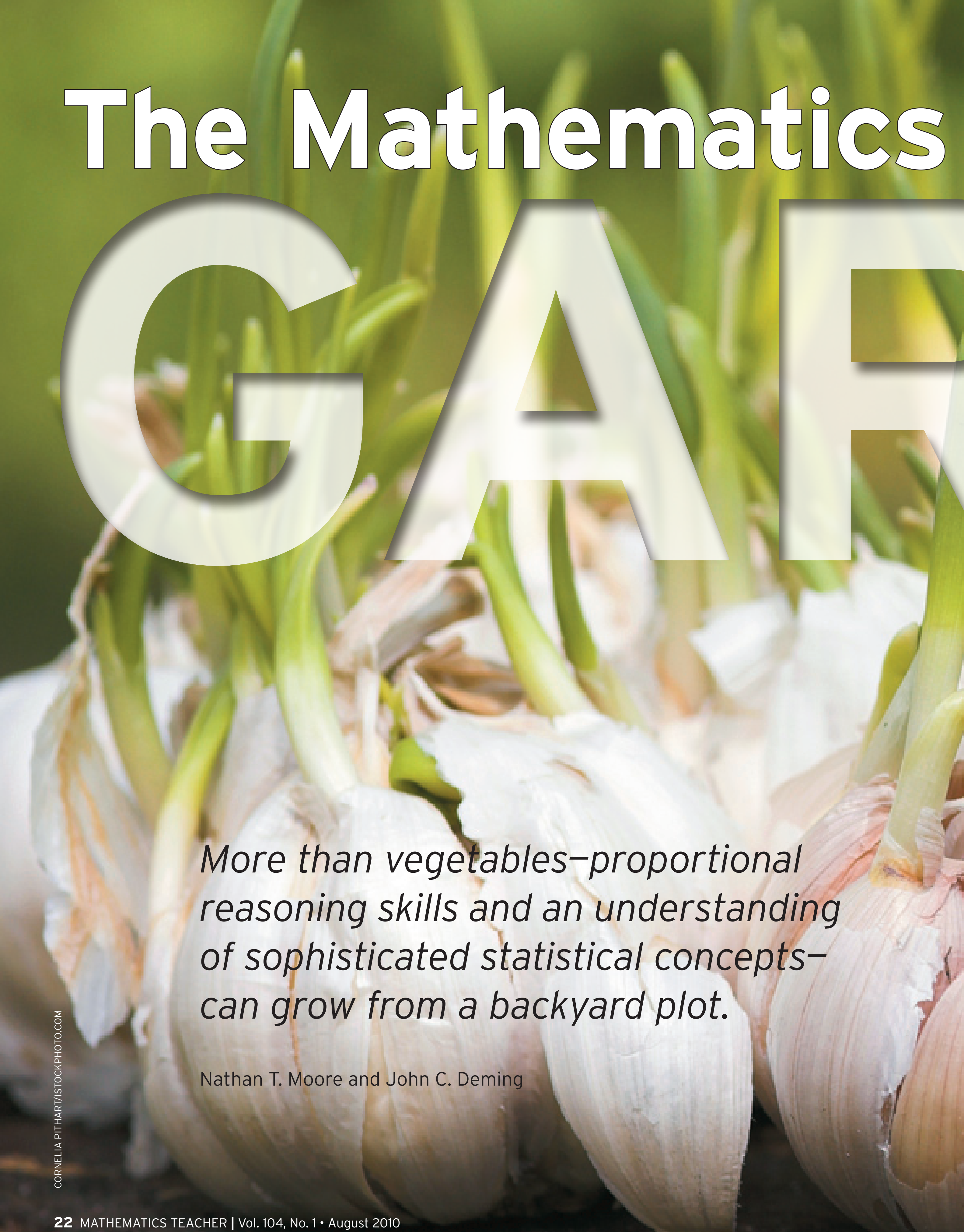

The Mathematics GAR

More than vegetables—proportional reasoning skills and an understanding of sophisticated statistical concepts—can grow from a backyard plot.

Nathan T. Moore and John C. Deming

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# of RLIC

In my household, it is embarrassingly common to be halfway through preparing dinner, with onions sautéing on the stove, and find that the bulb of garlic that was fine a week ago is now sprouting pale green shoots. The cloves of sprouted garlic are soft and bitter, and in the past I would have thrown the bulb away. But a few springs ago, as I was planting the vegetable garden, instead of throwing the sprouted bulb in the trash or on the compost heap, I planted it next to the lettuce, where it flourished.

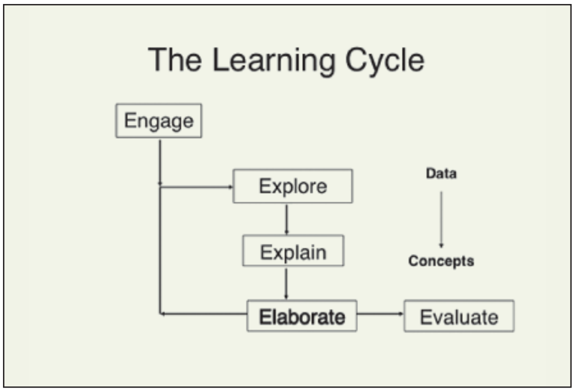

Fig. 1 The five-phase inquiry instructional approach is discussed by Lawson (2002) but uses the terminology from the Biological Sciences Curriculum Study (www.bscs.org).

Fig. 2 Planted in the late fall, garlic is one of the earliest spring vegetables. In this photograph, taken in late April, the shoots produced by individual cloves are well ahead of the foliage of other vegetables.

The problem presented here, which originated from the preceding vignette, develops several themes related to dimensional analysis and also introduces students to a few basic statistical ideas. I used this problem in a university preparatory chemistry class, designed for students with no chemistry background. However, this course is unique because one of the primary course goals is developing students' scientific thinking skills, including those described by Piaget (1964). These skills include control and exclusion of variables, classification, ratio and proportion, probability, combinatorial logic, the ability to formulate and manipulate mental models, and the ability to formulate and systematically test alternative hypotheses against data. These skills are used in all scientific disciplines and, accordingly, should be targeted in a secondary school mathematics classroom. But thinking skills are not discipline-specific. As student capacity develops, growth can be seen in similar fields (near transfer) and disparate fields (far transfer). For example, Adey and Shayer (1993) found that specific interventions in middle school science resulted in increased performance in science, mathematics, and English.

Philosophically, the course is organized in a constructivist framework. We purposefully choose learning environments that challenge students to develop concepts from data, so the teaching strategies we use follow the learning cycle format (Lawson 2002) (see **fig. 1**). The problem described here challenges students to apply their knowledge of measurement and proportions in a novel context (to the student, at least) and provides a place to practice probabilistic reasoning, a hard-to-target skill in physical science classrooms.

A BRIEF INTRODUCTION TO GARLIC

Typically, garlic is harvested in early to midsummer, when the green tops begin to yellow and wilt. The bulbs are dried for a few weeks and then ideally are stored at slightly above 0°C (Coleman 1995, p. 302) at 65% humidity for use during the following year. In the early fall, a fraction of this harvest is planted in a well-manured section of the garden, where each clove will set roots and perhaps produce a small green shoot. If properly mulched, garlic will survive the winter and send up green shoots well before anything else in the following spring.

Garlic is most easily managed if planted in a rectangular "patch" rather than in a traditional row (see **fig. 2**). I plant bulbs 4 inches apart in several dense rows, spaced 6 inches apart. (For more on this sort of dense vegetable cultivation, see Bartholomew [1981] or Adams [1988, pp. 82–89].) Over the spring, the plants will produce green shoots; later, some varieties will produce miniature seed heads, called *scapes*. (Both shoots and scapes are tasty, climatically "early" vegetables.) Scapes are essentially miniature garlic plants, and the vegetable can be propagated from year to year by planting these scapes.

Reproduction in garlic is predominantly asexual, meaning that the bulb produced by a planted clove contains many clones of the original clove. True "seed" production in garlic is quite rare; for more, see USDA (2004) or Kamenetsky and others (2004). Because of their small size, scapes can require two years to produce a harvestable bulb of garlic. The typical method of propagation is to simply break up a mature bulb of garlic and plant the large cloves. In this article, *garlic seed* refers to plantable cloves of garlic.

The main problem with saving seed from a fall harvest for spring planting is that crop failure is a fact of life. One year, I lost nearly all my garlic seed to mold when a week of rain thwarted its drying. However, the garlic sold in bulk at my local grocery store is quite viable, and I can buy sufficient seed to plant a large patch of cloves in my backyard.

THE GARLIC PROBLEM

Reluctant to buy too much seed and seeing garlic production as a concrete example for our students, we gave each student in the class a bulb of garlic (at a cost of about $8.00 for 20 bulbs) and presented the following problem:

> Suppose I want to grow enough garlic for cooking for the coming year and have enough left over for seed to plant in the fall.

- How much garlic should I purchase to have adequate seed?
- How big an area do I need to cultivate for this seed?
- How much money will I save by buying and growing plants rather than buying bulbs of equivalent quality at the supermarket?

The imaginative instructor will realize that these questions can generate lively discussion on a number of relevant topics, most of which can be described quantitatively. In some cases, students may voice questions that can lead to additional learning cycles. For example, does the size of the planted clove influence the number of cloves in the eventual bulb, and how does the variety of garlic correlate with the size of a clove? These questions provide opportunities for students to take more responsibility for their learning by challenging them to identify variables, form hypotheses, design experiments, and make predictions based on their hypotheses. And these opportunities give students a chance to do science.

Once students understood the context, we handed each student a bulb of garlic. We then gave the class about twenty minutes to study the bulbs and work on the problems given below.

The specific garlic we used in class was a red softneck of undetermined variety. In general, the size of the mature garlic bulb in July is proportional to the size of the clove planted the previous September, so not all the cloves in a given bulb would be worth planting. Our experience taught us that only cloves with diameter similar to or larger than that of a dime would have an acceptable yield. In our trials, this cutoff meant that by mass only about 91% of a bulb was usable for seed. The bulbs averaged 15.5 plantable cloves per bulb, with an average mass per plantable clove of about 3.5 grams.

Following are the specific questions we worked through with students, common student misconceptions, the class data set, and solutions.

To answer the problem's three questions, students weighed the bulbs, broke the bulbs into individual cloves, and decided what fraction of a given bulb would actually be useful for seed stock or for eating. With these data collected, we had each student write the measurements for his or her individual bulb on the class whiteboard. This data set is shown in **table 1**. The bottom rows of the table show the class averages. The fluctuations in the reported precision of mass measurements led to a discussion of significant figures. A typical bulb and a bulb broken into cloves are shown in **figure 3**.

How Much Garlic Should I Purchase to Have Adequate Seed?

In an average week, I use about 1 bulb of garlic for cooking. For a year's supply, I need to have about 52 bulbs of garlic available. Some garlic will undoubtedly be lost to spoilage. If we estimate the amount lost as 20% of the total (assuming that 80% of the harvest will be usable), we can calculate the portion needed for harvest using the following proportion:

$$\text{garlic not spoiled/garlic harvested} = 80\%/100\% = 0.8$$

This calculation implies that we will need 65 bulbs (52 bulbs/0.8). To have seed to plant in the following fall, we should also plan to have more than 65 additional cloves. Spoilage between harvest in June and planting in September will be lower than spoilage during the whole year. If we estimate 90% viability after three months, then, using logic similar to that above to plan next year's crop, we will need about 72 additional cloves at harvest time.

Students' data indicated that the average yield of usable cloves per bulb was about 15.5. Therefore, 5 extra bulbs, for a total of 70 bulbs, would probably provide sufficient garlic for a year's consumption, planting, and loss.

Extension

An interesting learning cycle that clove health might generate would involve the students dividing cloves into healthy, diseased, and questionable groups and then planting these groups in a school garden in the fall. When the cloves sprout in the following spring (near the end of the academic year), students could test their predictions. Additional predictions that could be tested in a class might include the germination ratio as a function of clove mass, garlic variety, mulch depth, and so on.

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Table 1 Garlic Data Collected in Class					
Student Number	**Bulb Mass (g)**	**Number of Plantable Cloves**	**Plantable Cloves Mass (g)**	**Number of Unplantable Cloves**	**Unplantable Cloves Mass (g)**
1	57.152	16	54.832	0	2.320
2	70.757	16	62.632	5	8.125
3	51.712	17	49.059	1	2.653
4	55.136	16	50.573	2	4.563
5	57.214	17	46.220	7	10.994
6	63.071	15	54.421	4	8.650
7	60.000	16	58.180	0	1.820
8	53.326	14	49.086	4	4.240
9	59.200	15	56.970	0	2.230
10	66.890	12	61.956	5	4.934
11	61.290	14	53.330	3	7.960
12	59.568	16	57.986	2	1.582
13	71.088	14	59.967	4	11.121
14	62.299	13	49.193	6	13.106
15	60.438	21	58.751	7	1.687
Average	60.609	15.467	54.877	3.333	5.732
Median	60.000	16	54.832	4	4.563
Standard Deviation	5.674	2.100	5.154	2.440	3.935

How Big an Area Do I Need to Cultivate?

In addition to the planting instructions already provided, we told students that the planned garlic patch was tentatively a 2 foot × 15 foot section of soil. If we assume that the rows run the "long" way down the patch and that the bulbs are planted every 4 inches along the row, students can use dimensional analysis to calculate the number of cloves that can be planted in a single row:

$$(15 \text{ ft./row})(12 \text{ in./ft.})(1 \text{ clove}/4 \text{ in.}) = 45 \text{ cloves/row}$$

Subsequent discussion allows students to connect their proportional reasoning to the "common sense" layout of the garden. The number of rows that can be planted in a 2-foot width is determined by the way the rows are laid out. If, for example, the outside rows run at the absolute edge of the bed, one could fit 5 rows of garlic in the bed. However, horticulturally speaking, the foot traffic and lack of cultivation at the edge of the bed would probably lead to a significantly lower yield in the two outside rows. Better yields might be obtained by planting 4 rows and allowing a 3-inch buffer on either side of the outside rows.

If we plant 4 rows, with 45 cloves per row, the patch could hold about 180 garlic plants—probably more than enough for a family's annual needs.

How Much Money Will I Save by Buying and Growing Plants?

Looking at the data, we see that the average bulb of garlic has a mass of 60.6 grams. If we want to splurge and plant 180 cloves (the maximum that our garlic patch will allow), the number of cloves needed can be calculated using dimensional analysis:

$$(180 \text{ cloves})(1 \text{ bulb}/15.5 \text{ plantable cloves})(60.6 \text{ g/bulb})(1 \text{ lb.}/454 \text{ g}) \approx 1.55 \text{ lbs. of garlic}$$

Locally, garlic sells for \$2.69 per pound, so our seed would cost about \$4.17. If 90% of the plants survived and 20% of the garlic was lost in storage, the 180 plants would become $180 \cdot 0.9 \cdot 0.8 \approx 130$ bulbs of garlic. If we assume that this garlic is simi-

NATHAN MOORE

Fig. 3 An intact bulb and cloves broken out from a second bulb. All the cloves are probably plantable, although four cloves are near the "dime" threshold size used in class.

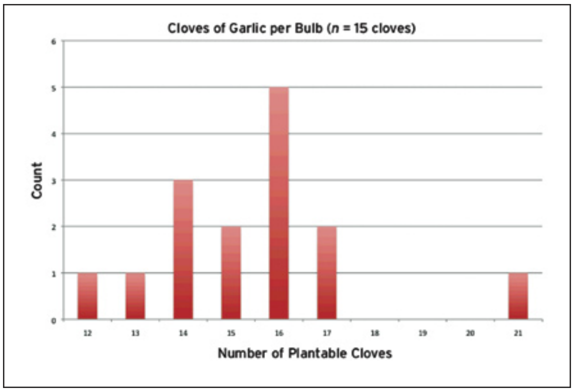

Fig. 4 To summarize students' individual observations, the class as a whole constructed a histogram of plantable cloves per bulb of garlic and used it to explain the meaning of average number of plantable cloves seen per bulb (15.5 cloves) and the standard deviation about that average (2.1).

lar to the seed stock, the mass of 130 bulbs would be as follows:

$$(130 \text{ bulbs})(60.6 \text{ g/bulb})(1 \text{ lb.}/454 \text{ g}) \approx 17.4 \text{ lbs.}$$

The calculation for the amount of garlic a family might buy from the grocery store annually is

$$(52 \text{ bulbs/yr.})(60.6 \text{ g/bulb})(1 \text{ lb.}/454 \text{ g.}) \approx 6.9 \text{ lbs.}$$

There are several things to note about this calculation. First, at grocery store prices—\$2.69 per pound—1 bulb a week is about \$18.56 worth of garlic over a year, so growing your own garlic is slightly more economical. Second, the organic "heirloom" garlic at my local food co-op sells for upward of \$4 per pound, and the vegetables you produce in your backyard can certainly measure up to this quality. If you were to buy the projected 17.4 pounds that the garlic patch produces, the cost would be nearly \$70, so if food quality is your concern, growing your own garlic is not an absurd idea. Students can also conduct a number of similar comparisons along these lines—such as cost per seed, average yield per seed, and so on.

ASSESSMENT

When most of the students were nearly finished with these problems, we presented them with this follow-up question:

> I have a few bulbs of garlic left over that have not been opened. How many cloves would you guess are in one of those bulbs? About how much would one clove weigh?

The goals of this lesson, as mentioned earlier, include developing students' scientific thinking skills as well as their content knowledge. Therefore, the question regarding how many cloves are in one bulb is critical because it challenges students to apply probabilistic reasoning, a scientific thinking skill often neglected in physical science classrooms. Many students used the class average of cloves per bulb to predict the number of cloves in the next bulb. However, we specifically asked students to describe a reasonable number or range of cloves to expect, thus forcing them to be more specific in their rationale for a given prediction.

To come to a collective agreement, in class we summarized the students' discussion by creating the histogram shown in **figure 4**. To make this figure, we computed how many times an exact number of plantable cloves was found in a bulb of garlic. As the data show, the most common—and, we argued, the most probable—outcome was 16 plantable cloves. In talking about a range of possible outcomes, we can then say that 5 times out of 15, or 1/3 of the time, students in the class saw this outcome. Further, the range of 15 to 17 plantable cloves accounts for 9 of the 15 outcomes—almost 2/3 of the total probability. Finally, the range of 14 to 17 plantable cloves accounts for 12 of the 15 outcomes—80% of the total.

Therefore, since 12 of the 15 total outcomes correspond to 14 to 17 plantable cloves per garlic bulb, we can say with probability of 12/15 = .75 that another bulb of garlic from the same produce bin at the grocery store would contain 14 to 17 plantable cloves.

This process of building a logical argument makes possible the transition of language to a "probability distribution" or "probability density," a continuous curve of likelihood that is suggested by students' measurements. A plot of probability density and associated students' measurements are included in **figure 5**. The area under this probability density curve is computed to give the net probability of a given outcome. Of course, the number of bulbs sampled is quite low, so the data are plotted

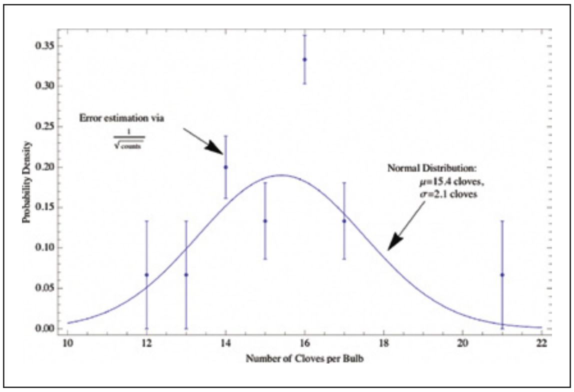

Fig. 5 This graph contains the same data as **figure 4** but with a vertical axis rescaled to plot probability density. Given the suggestive nature of the data, the figure also contains the "normal" (Gaussian) probability density distribution, with standard deviation and mean as determined in **table 1**.

with a Poisson-weighted error bar. If the exercise were repeated over several different classes (with $n \approx 100$ bulbs), the data would likely be of sufficient quality to justify representation with a continuous distribution.

In class, the validity of this prediction was further validated when the two remaining bulbs contained, respectively, 16 and 15 plantable cloves. This outcome seemed to strengthen further the power of the statistical argument in the students' minds.

CONCLUSION

Challenging students to apply course concepts and thinking skills in new contexts is fundamentally important. Speaking simply, if our students cannot apply what they have learned to a new situation, can we say that they have learned anything? Taking one class period of chemistry and devoting it to a nonchemistry topic fulfils our goal (described in Deming and Cracolice 2004) to teach students *how* to think rather than teach them *what* to think. Therefore, providing students with a less-familiar—or far-transfer—type of problem challenges them to take data and make sense of those data to arrive at a plausible answer.

These are the skills necessary to become successful in college mathematics and science classrooms. They warrant a certain amount of educational time, even at the expense of a few class periods of traditional content.

ACKNOWLEDGMENTS

This work was supported in part by the Minnesota State Colleges and University System's Center for Teaching and Learning Instructional Development Grant Program. Most of the units were developed for high school chemistry in the Frameworks for Inquiry program at the University of Montana, as part of a Mathematics and Science Partnership Grant administered by the Montana Office of Public Instruction.

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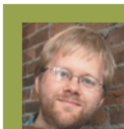

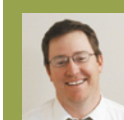

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